The Impact of Connection Topology and Agent Size on Cooperation in the Iterated Prisoner's Dilemma

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Abstract—This study revisits earlier work, concerning the evolutionary trajectory of agents trained to play iterated prisoner's dilemma on a combinatorial graph. The impact of different connection topologies, used to mediate both the play of prisoner's dilemma and the flow of genes during selection and replacement, is examined. The variety of connection topologies, stored as combinatorial graphs, is revisited and the analysis tools used are substantially improved. A novel tool called the *play profile* summarizes the distribution of behaviors over multiple replicates of the basic evolutionary algorithm and through multiple evolutionary epochs. The impact of changing the number of states used to encode agents is also examined. Changing the combinatorial graph on which the population resides is found to yield statistically significant differences in the play profiles. Changing the number of states in agents is also found to produce statistically significant differences in behavior. The use of multiple epochs in analysis of agent behavior demonstrates that the distribution of behaviors changes substantially over the course of evolution. The most common pattern is for agents to move toward the cooperative state over time, but this pattern is not universal. Another clear trend is that agents implemented with more states are less cooperative.

I. INTRODUCTION

MODELING cooperation and conflict with the iterated prisoners dilemma (IPD) has shown that cooperation has many possible sources. In this study we compare different abstract connection topologies, encoded as combinatorial graphs, to see what impact they have on the behavior of agents being trained to play the iterated prisoner's dilemma. An earlier study [1] examined populations evolved on thirteen graphs with 32 and 64 vertices from five families of regular graphs that limited mating to determine which were more cooperative. Experiments were also done in which the graphs did or did not limit which agents were played when evaluating skill at IPD. This study extends and enlarges the earlier one and uses more sophisticated analysis developed in the meanwhile. Several of the analysis tools appear in [12].

Prisoner's Dilemma [14], [13] is a classic model in game theory. Two agents each decide, without communication, whether to cooperate (C) or defect (D). The agents receive individual payoffs depending on the actions taken. The payoff for mutual cooperation C is the *cooperation* payoff. The payoff for mutual defection D is the *defection* payoff. The two asymmetric action payoffs S and T, are the *sucker* and *temptation* payoffs, respectively. In order for a two-player

simultaneous game to be considered prisoner's dilemma, it must obey the following inequalities:

$$S \le D \le C \le T \tag{1}$$

and

$$(S + T) \le 2C. \tag{2}$$

The iterated version of the game is widely used to model emergent cooperative behaviors in populations of selfishly acting agents and is often used to model systems in biology [28], sociology [22], psychology [26], and economics [20]. Many researchers have investigated the evolution of prisoner's dilemma playing agents [24], [19], [18], [27], [17], [23], [16], [25] with a focus on understanding the evolution of cooperation, particularly in changing environments. Another study that treats the problem of spatially structured iterated games appears in [15] while [21] examines both spatial structure and the critical issue of representation.

II. MATHEMATICAL BACKGROUND

Some familiarity with graph theory is assumed. An excellent reference in the area is [29]. The theory required in this study is reviewed here. A *combinatorial graph* or *graph G* is a collection $V(G)$ of vertices and $E(G)$ of edges where $E(G)$ is a set of unordered pairs from $V(G)$. Two distinct vertices of the graph are *neighbors* if they are members of the same edge. The *neighborhood* of a vertex is the set of all neighbors of that vertex. The number of edges containing a vertex is the *degree* of that vertex. If all vertices in a graph have the same degree, then the graph is said to be *regular*. If the common degree of a regular graph is k, then the graph is said to be k-*regular*. A graph is *connected* if one can go from any vertex to any other vertex by traversing a sequence of vertices and edges. The *diameter* of a graph is the largest number of edges in a shortest path between any two of the vertices. The diameter is the maximum distance, in terms of shortest paths, between any two vertices in the graph.

A. Graphs Used

The following is a list of the graphs used. Examples of graphs of this type, sometimes with different parameters for viewability, are shown in Figure 1.

1) The **complete graph**, K_n, has n vertices and all pairs of vertices have an edge between them.
2) In a **cycle** on n vertices $V = \{0, 1, ..., n-1\}$ the edge set consists of all edges of the form $\{i, i+1\}$ (mod n), where $i \in V$.
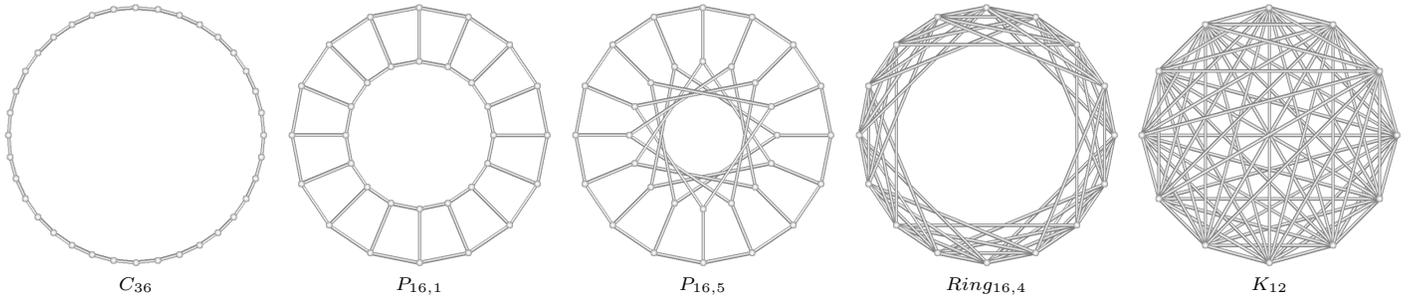
Fig. 1. Examples of graphs of the types used in this study.

Name	Index Name	Size	Diameter	Radius	Edge Density	Mean Degree
C_{36}	C36	36	18	18	0.0571	2
C_{72}	C72	72	36	36	0.0282	2
$P_{16,1}$	P16,1	32	9	9	0.0968	3
$P_{32,1}$	P32,1	64	17	17	0.0476	3
$P_{16,5}$	P16,5	32	6	6	0.0968	3
$P_{32,5}$	P32,5	64	7	7	0.0476	3
$Ring_{36,4}$	R36,4	36	5	5	0.229	8
$K36$	K36	36	1	1	1	35
$K72$	K72	72	1	1	1	71

3) In a **generalized Petersen graph** $P_{n,k}$ with parameters n and k (where $k \leq n$), and vertex set 0, 1, . . . , 2n-1. The vertices 0, . . . , n-1 are connected in a standard n-cycle while the vertices $n, ..., 2n-1$ are connected with the $(n+i)$th vertex connected to the $(n+i+k)$th (mod n) vertex, where $0 \leq i \leq n-1$. Finally, pairs of vertices $i, n+i$ are also connected.

4) In an **(n,k)-ring** with vertex set $\{0, 1, ..., n-1\}$ the edge set consists of all edges of the form $\{i, i+j\}$ (mod n), where $i \in V$ and $j \in \{1, ..., k\}$.

Table I lists the graphs used in this study together with their parameters.

III. MODEL SPECIFICATION

The prisoner's dilemma payoffs used in this study are $S = 0$, $D = 1$, $C = 3$, $T = 5$. These values are chosen for consistence with the earlier study.

A. Agent Representations

The finite state machines (FSMs) used in this study use the Mealy architecture with responses encoded on the transitions (the Moore architecture encodes responses on the states). An example of the type of FSMs used in this study appears in Figure 2. A finite state machine is always driven by the opponent's last action. For this reason the resource we vary is the number of states available to the machine. In [2] it was discovered that the behavior of finite state machines playing a collection of different 3-move games changed substantially between 8 and 80 state machines, except when the game being played was zero-sum. As Prisoner's dilemma

is not zero-sum, we study the impact of varying the resource "number of states" for FSM agents in the IPD.

C→5		8 states
State	If C	If D
0	C→6	C→ 4
1	D→4	C→ 5
2	D→1	D→ 5
3	D→3	D→ 2
4	C→2	D→ 3
5	D→3	C→ 7
6	C→2	C→ 3
7	C→4	D→ 2

Fig. 2. An automata of the sort used as the agent representation in this study. This automata has eight states, cooperates initially, and uses state 5 as its initial state.

A recent study [5] showed that the number of states in a finite state agent can impact its chance of becoming cooperative and so experiments are replicated three times with agents having 8, 24, and 72 states. The earlier study on the impact of connection topologies on cooperation used agents with eight states. The agent sizes 24 and 72 permit exploration with moderate and large numbers of states. The variation operators used are two point crossover of the array of states, with the choice of initial state and action attached to the first state, and a mutation operator that operates as follows: machine's initial state or action changes 5% of the time each, transition changes 40% of the time, and a response changes 50% of the time. This mutation operator was retained from previous studies for consistency.

B. The Evolutionary Algorithm

The evolutionary algorithm evolves agents on a combinatorial graph. Fitness is evaluated by computing the average score when an agent plays 150 rounds of IPD against a set of other agents. The *local fitness* experiments use agents in the neighborhood of a vertex. The *universal fitness* experiments use the entire population to evaluate fitness. Once fitness is evaluated, a population updating is performed. The bottom third of the population is replaced and, again, there are local and universal variants of the experiment with the local version limiting reproductive partners to graph neighbors and

the universal version drawing on the entire population. In both versions, the following procedure is used to update agents.

Fitness proportional selection is used to choose a partner from those available. The agent being replaced undergoes crossover with the partner (the crossover is one-sided and does not affect the partner). The resulting new agent is then subject to a mutation of the sort described above. Once all the agents have been updated, the algorithm continues to the next fitness evaluation.

IV. Design of Experiments

In each experiment the evolutionary algorithm was run for 800 generations using 100 replicates (experiments with distinct random number seeds). The elite portion of the population in generations 50, 100, 200, 400, and 800 was saved for analysis. This yields 100 sets of 24 machines at each of five epochs. A number of descriptive statistics are saved in each generation of each replicate. These include the mean fitness and the variance of fitness.

Each graph listed in Table I was tested for three cases:

1) Mating is limited to neighbors on the graph but play occurs between all individuals.
2) Play is limited to neighbors on the graph but mating occurs between all individuals.
3) Both play and mating are limited to neighbors on the graph.

The complete, cycle, and Petersen graphs were all tested with 8, 24, and 72 state agents for each of the above cases, but due to computational time constraints it was decided to test the ring graph using only 24 states for each of the three limitations on fitness evaluation and reproduction. Thus, there was a total of 75 different experiments.

1) Play Profiles: One of the primary assessments of an evolutionary system for training prisoner's dilemma agents changes is the probability that a given population is co-operative. In the past studies [11], [9], [3], [4], [6], it was established that when 150 rounds of iterated prisoner's dilemma with the 0, 1, 3, 5 payoff scheme are used in fitness evaluation, an average score of 2.8 signifies that FSMs end up in a cycle of sustained cooperation. We extend this fitness measure in this study, a technique also used in [5]. The average fitness of a population is a sum of pairs of payoffs with one of three values: (1,1), (0,5), or (3,3). This places the mean value in the range $1 \leq \mu \leq 3$. We divide this region into ten equal intervals, the top one corresponding to the definition of cooperation given above. At each epoch we record the number of populations in each of the ten intervals. The resulting 5×10 table is the *play profile* for an experiment. This is a novel method of assessing the behavior of IPD agents first introduced in [5]. An example of a play profile is given in Figure 3.

V. Results and Discussion

For each of the 75 trials, a chi-square test was used to compare the play profiles of each trial with the same number of states. The null hypothesis of no difference was rejected for p-values < 0.05, otherwise the null hypothesis was not rejected and it could not be concluded that the play profiles were different. Furthermore, it was determined that a p-value between 10^{-6} and 0.05 would be considered significant, while any p-value $\leq 10^{-6}$ would be considered highly significant.

Along with the same-state comparisons, one set of trials with each of the three limitation cases was randomly selected from each connection topology and used to compare the differences in play profile across automata with different numbers of states, for a total of eight cross-state trial sets. In total, this resulted in 1004 pairwise comparisons. A summary of the number of results that fell into each of highly significant difference, significant difference, and no significant difference can be found in Table II.

TABLE II
SUMMARY OF THE NUMBER OF PAIRS THAT FALL INTO EACH LISTED CATEGORY. SAME STATE COMPARISONS ARE BETWEEN AGENTS IMPLEMENTED WITH THE SAME NUMBER OF STATES. CROSS STATE COMPARISONS COMPARE AGENTS EVOLVED WITH THE SAME CONNECTIVITY AND RESTRICTIONS BUT DIFFERENT NUMBER OF STATES.

	Highly Significant Difference	Significant Difference	No Significant Difference	Total
Same State	428	142	398	978
Cross State	16	5	3	24

Among the same-state trials, approximately 58% of tested pairs were shown to have statistically significant differences in their play profiles. Furthermore, approximately 44% were shown to have highly significant differences in their play profiles. This is strong evidence that the connection topologies have a significant affect on the play profile. Among the cross-state trials, approximately 88% of tested pairs were shown to have statistically significant differences in their play profiles with approximately 67% demonstrating highly significant differences. This provides strong evidence that the number of states significantly affects the play profile.

A. Impact of Changing Connection Topology

In comparing the cycle C_{36} graph and the complete K_{36} connection topologies, with the limitation that individuals could only mate and play with their neighbors, it was found that the play profiles could be shown to be different with a high degree of significance. In this case, the p-values were found to be $p_8 = 1.60 \times 10^{-4}$, $p_{24} = 6.22 \times 10^{-9}$, and $p_{72} = 6.54 \times 10^{-23}$. See Figure 4 for the play profiles of this case.

By examining the complete K_{36} graph and the Petersen $P_{16,1}$ connection topologies, with the limitation that individuals could only mate with their neighbors, it was found that the play profiles could be shown to be different with a high degree of significance. In this case, the p-values were
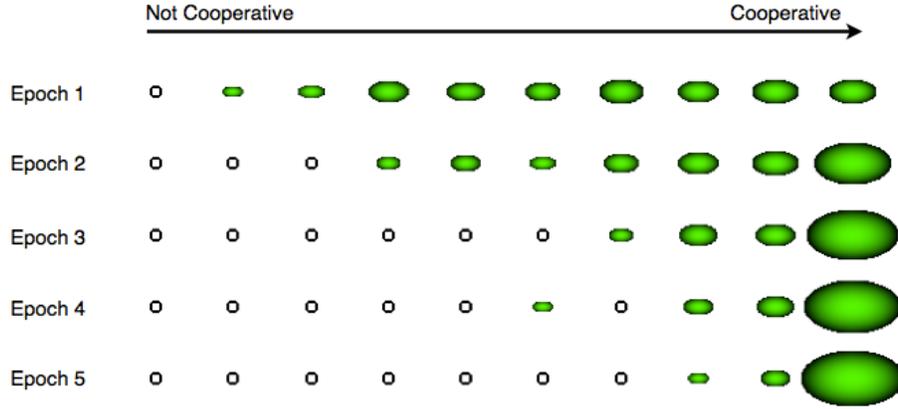
Fig. 3. In each play profile the size of the green oval is reflective of the number of replicates within each average score interval, 1.0-1.2, 1.2-1.4, $\cdots$, 2.8-3.0. These intervals range from least cooperative on the left to fully cooperative on the right. Each row represents each tested epoch for generation 50, 100, 200, 400, and 800.

found to be $p_8 = 1.72 \times 10^{-42}$, $p_{24} = 1.86 \times 10^{-9}$, and $p_{72} = 1.59 \times 10^{-23}$. See Figure 5 for the play profiles of this case.

Further examination shows that for the cycle C_{72} graph and the Petersen $P_{16,5}$ connection topologies, with both play and mating limited by the graphs, it was found that the play profiles could be shown to be different with a high degree of significance. In this case, the p-values were found to be $p_8 = 1.68 \times 10^{-100}$, $p_{24} = 1.83 \times 10^{-4}$, and $p_{72} = 5.46 \times 10^{-76}$. See Figure 6 for the play profiles of this case.

Overall, it is readily apparent that different connection topologies yield significantly different play profiles. Limiting only play with the graph seems to yield the fastest evolution of cooperation; this may be the result of having a more stable fitness function (graph neighborhood) while having full access to genetic diversity. Further research will be needed to determine any pattern that illuminates which types of connection topologies yield what type of play profiles. No strong correlation was found between the graph parameters listed in Table I and the character of play profiles.

B. Impact of Changing Number of States

Besides comparing the results achieved from different graph topologies, we also wished to examine the effect of varied state sizes. From the eight cross-state trial sets, it was shown that changing the number of states produces differences in the play profile that are statistically significant. For the complete graph K_{36} tested with the condition that mating could occur with anyone but only neighbors might play each other, it was found that the play profiles produced by the 8-state automata, 24-state automata, and 72-state automata were all significantly different, with p-values of $p_{8,24} = 5.53 \times 10^{-4}$, $p_{24,72} = 1.66 \times 10^{-31}$, and $p_{8,72} = 2.92 \times 10^{-65}$. It can be seen that as the difference between the number of states increased, the significance also increased, *i.e.* the p-value for $p_{8,72} << p_{24,72} << p_{8,24}$. See Figure 7 for the play profiles.

For the Petersen graph $P_{32,5}$ tested with the condition that

play could occur with anyone but only neighbors might mate with each other, it was found that when comparing the play profiles produced by the 8-state automata, 24-state automata, and 72-state automata using a chi-square test, the p-values were as follows: $p_{8,24} = 0.0578$, $p_{24,72} = 4.43 \times 10^{-4}$, and $p_{8,72} = 1.51 \times 10^{-13}$. Thus, while the difference between the 8-state and 24-state automata was not statistically significant, the inequality from the previous example still holds. See Figure 8 for the play profiles.

Seven of the eight sets of comparisons across trials with different numbers of states also demonstrated that the greater the difference between the number of states, the more significant the difference between the two play profiles. This result demonstrates the importance of the number of states used, as selecting an alternate number of states could yield significantly different results.

Furthermore, as the number of states increased, so too did the amount of spread among the different levels of cooperativeness. In most cases, the 72-state automata resulted in the least cooperative scores and, in general, the 8-state trials produced the most cooperative results. Refer to Figure 7. Even in cases where this effect was less pronounced, it was usually most apparent in the early epochs and decreased over time. Refer to Figure 8. Because of the increase in spread, the overall level of cooperativeness was reduced in these cases. The only exception to this rule was in the case where the graph topology of the cycle C_{72} limited both play and mating. See Figure 6. In this case an inverse relationship occured. It is not understood why this inversion occurs, but this trial was repeated to rule out human error. Further study will be required to gain an understanding of this odd behavior.

C. Impact of Changing Limitations from Graph Topology

To examine the importance of the various playing and mating limitations used in this experiment, it was necessary to compare the same graph, with the same number of states, but with each of the three cases for topological limitation.

In comparing the cycle C_{36} graph with 24 states, it was found that the play profiles could be shown to be different with a high degree of significance. Here we will denote the limitation of play to neighbors as "LP", the limitation of mating to neighbors as "LM", and the limitation of both as "LB". In this case, the p-values were found to be $p_{LP,LM} = 1.32 \times 10^{-26}$, $p_{LP,LB} = 6.22 \times 10^{-9}$, and $p_{LM,LB} = 7.60 \times 10^{-13}$. See Figure 10 for the play profiles of this case.

Finally, some of the most significant differences found were those in which two trials with different graph types and different topological limitations were compared. By comparing the cycle C_{72} graph with the limitation that individuals could only mate with their neighbors and the Petersen $P_{32,5}$ graph with the limitation that individuals could only play against their neighbors, it was found that the play profiles could be shown to be different with an extremely high degree of significance. In this case, the p-values were found to be $p_8 = 1.09 \times 10^{-134}$, $p_{24} = 2.11 \times 10^{-66}$, and $p_{72} = 2.61 \times 10^{-61}$. See Figure 11 for the play profiles of this case.

Overall, limiting mating to neighbors while allowing for play with anyone results in little spread among the various bins, but a strong shift from less cooperative to more cooperative over time. See Figure 8. Furthermore, by limiting play, a general trend of greater spread amongst bins to near-universal cooperation arises. Refer to Figure 9.

D. Impact of Graph Parameters on Cooperation

Based on the results of this study, there are no strong correlations between any of the graph parameters and the level of cooperation. A slight positive correlation between the number of nodes and the level of cooperation was found, except in the case of C_{36} and C_{72}, where C_{72} remains the exception. A small positive correlation was also found between the edge density and the level of cooperation. Refer to Figures 10 and 11 where the $R_{36,4}$ graph has a significantly higher edge density. While the 'Limit Play' case shows no significant difference, the 'Limit Mate' and 'Limit Both' cases are significantly different with $p = 9.01 \times 10^{-16}$ and $p = 4.94 \times 10^{-7}$, respectively. No other clear conclusions could be made regarding the relationship between the graph parameters and the level of cooperation. Further research will be needed to determine if any relationships exist and what they may be. This includes both running more types of graph to strengthen the signal and looking for other graph invariants that yield a higher correlation with cooperativeness.

VI. CONCLUSIONS AND NEXT STEPS

The results of this study show that the structure of the connection topologies have a large impact on the play profile generated by an evolutionary algorithm training agents to play the iterated prisoner's dilemma. This means that the connection topology is an important factor in the emergence of cooperation. This study also demonstrates that the impact of the number of states an agent is allocated has a large impact in prisoner's dilemma on graphs, echoing and extending the result from [5].

While the research is still in early stages, the results presented in this study could eventually be used to help set up the layout of an office or classroom to encourage cooperation. In such an application, the mating events would reflect social learning and/or training in the workplace, while play would reflect the daily interactions and working together that occur in school and the workplace. Alternatively the results could be used to organize human interactions within a structure like a social network, or a hierarchical command structure like in the military. Any network system that involves teamwork in some way could benefit from this type of research via insight into simple, fixed cost control strategies available whenever connection topologies can be controlled.

Realizing these applications will require substantial additional research. Analysis of the impact of network parameters, the relationship between the various graph structures, and the resulting level of cooperation is needed. For the purpose of this study, we focused on the differences between the play profiles generated by modifying various parameters and graph types. In future work, it would be beneficial to examine the specific fitness - and thus cooperation level - of each type of graph to determine if a certain graph tends to produce more cooperative players. We can see already that both time of evolution and graph connectivity control migration of the population toward the right-most bins, reflective of higher levels of cooperation. In future work we would determine the specific parameters and trends that promote such migration.

Confirmation that the number of states in each automata has a clear affect on the outcome of the trial re-emphasized that it is imperative that the number of states, or more generally the agent representation [7], used in an agent based game theory experiment be reported, and that researchers be aware that current results may not hold if the number of states, or other aspects of the agent representation, are changed. This is a critical result for the field of evolutionary computation as it highlights an issue that has not been examined much in previous literature.

A. More Graphs, More Parameters

As in the earlier study [1], the graphs used in this study were highly symmetric and regular. This was intentional to avoid making too many changes in a single step in a research program. This study both implements play profiles and checks the impact of changing the number of states used in an agent. A clear next step is to use asymmetric graphs, irregular graphs, random graphs, and generally, more graphs. Clear candidates include trees and random graphs with different models including edge probability or formation via addition or random cliques.

Given that cycles were behavioral outliers in this study, the *girth* of graphs may be an important parameter. The girth of a graph is its shortest cycle. An example of a graph with high girth, given its regular degree and number
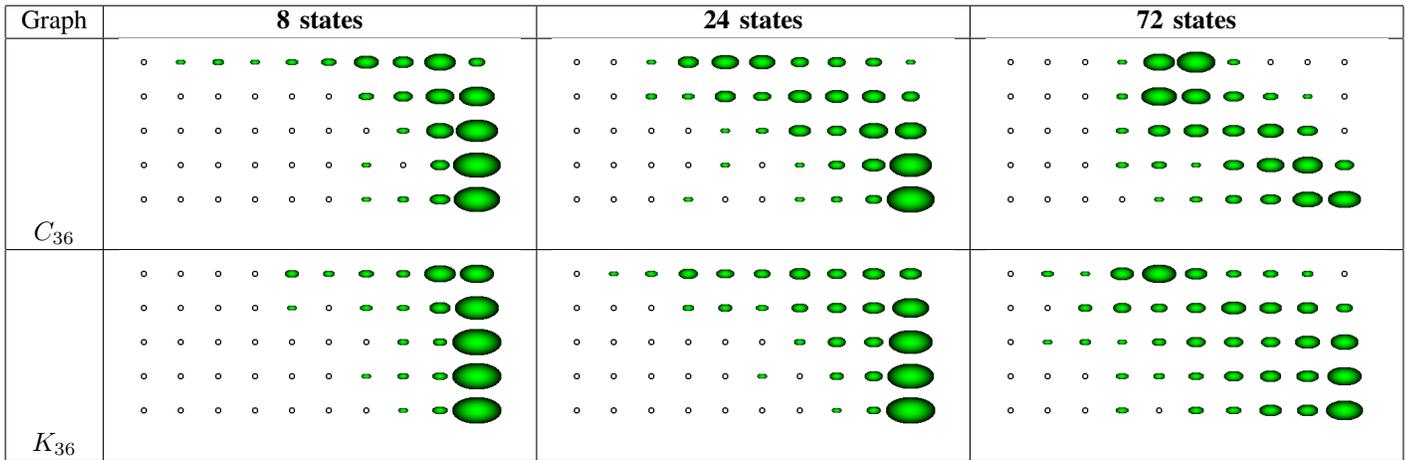
Graph	**8 states**	**24 states**	**72 states**

Fig. 4.   Play profiles for 8, 24, and 72 state machines evolved on different graphs with the graphs limiting both play and mating.
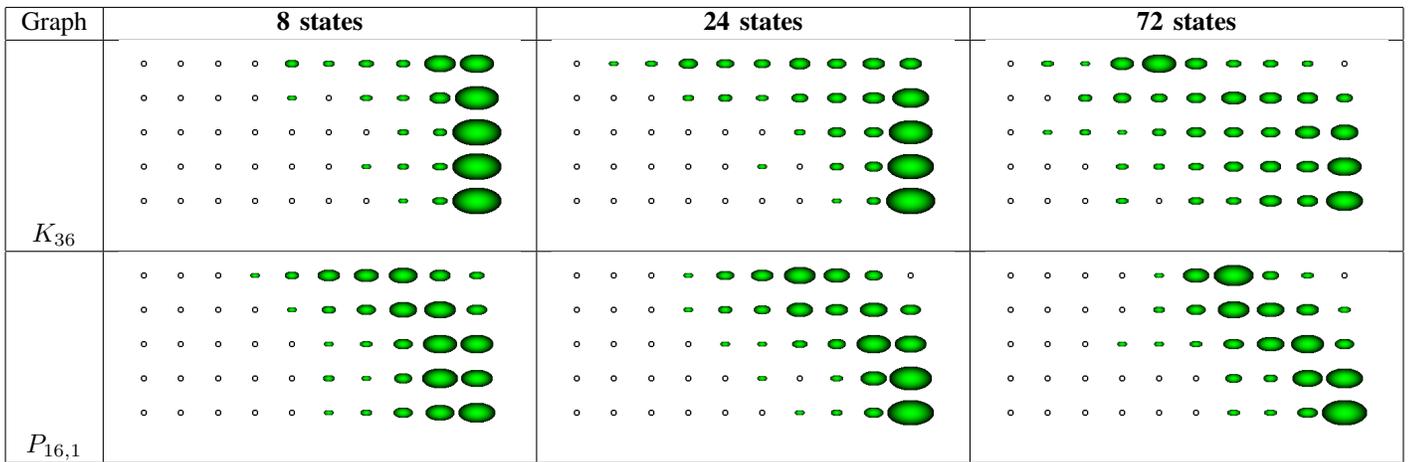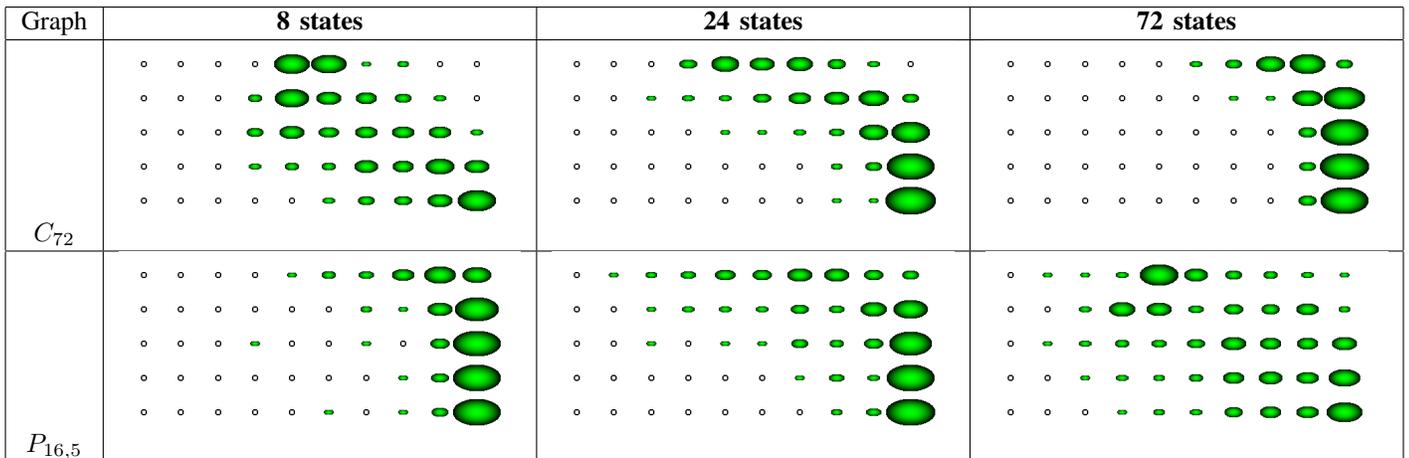
Graph	**8 states**	**24 states**	**72 states**

Fig. 5.   Play profiles for 8, 24, and 72 state machines evolved on different graphs with the graphs limiting mating behavior.

Graph	**8 states**	**24 states**	**72 states**

Fig. 6.   Play profiles for 8, 24, and 72 state machines evolved on different graphs with the graphs limiting both play and mating.
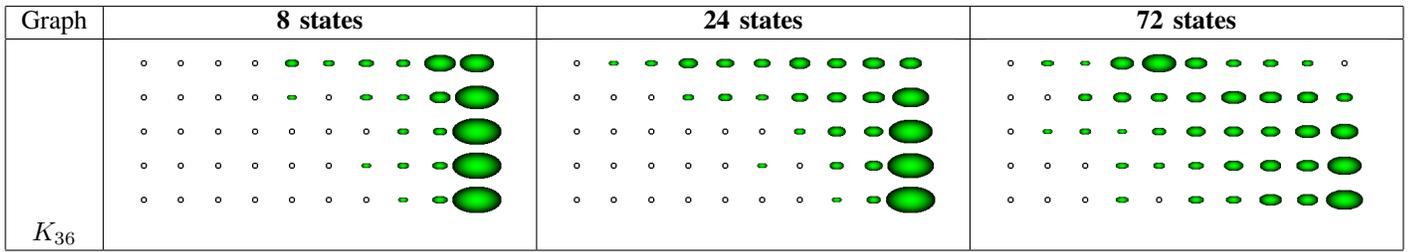
Graph	8 states	24 states	72 states
K_{36}			

Fig. 7. Play profiles for 8, 24, and 72 state machines evolved on the complete graph K_{36} with the graph limiting mating.

Graph	8 states	24 states	72 states
$P_{32,5}$			

Fig. 8. Play profiles for 8, 24, and 72 state machines evolved on the Petersen graph $P_{32,5}$ with the graph limiting mating.

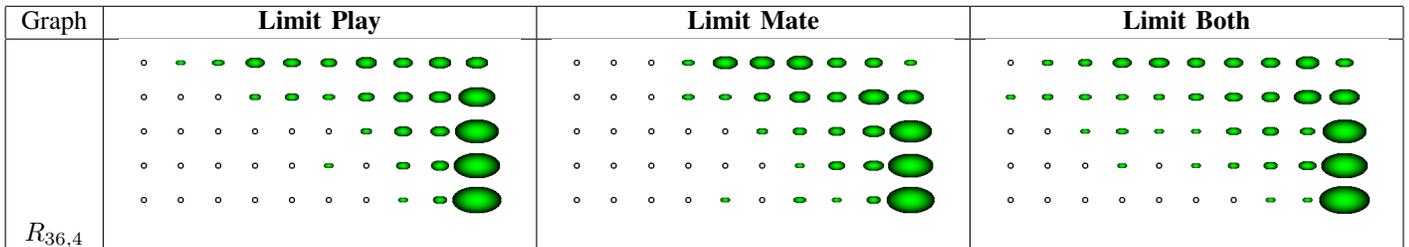| Graph	Limit Play	Limit Mate	Limit Both
$R_{36,4}$			

Fig. 9. Play profiles for 24 state machines evolved on the ring $R_{36,4}$ graph with various limiting behaviours.

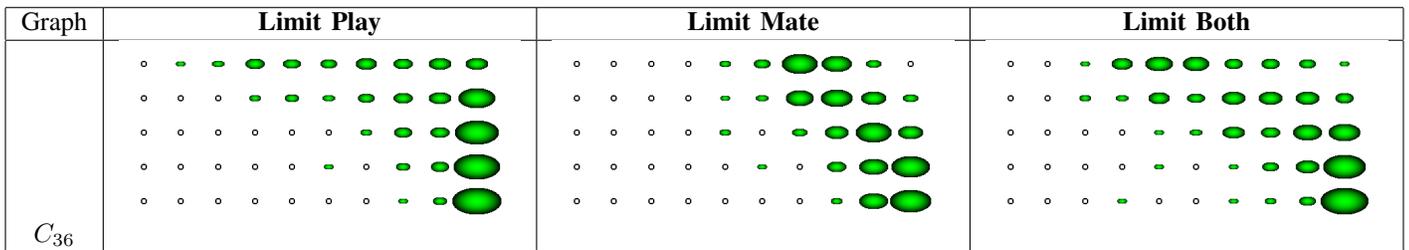| Graph	Limit Play	Limit Mate	Limit Both
C_{36}			

Fig. 10. Play profiles for 24 state machines evolved on the cycle C_{36} graph with various limiting behaviours.

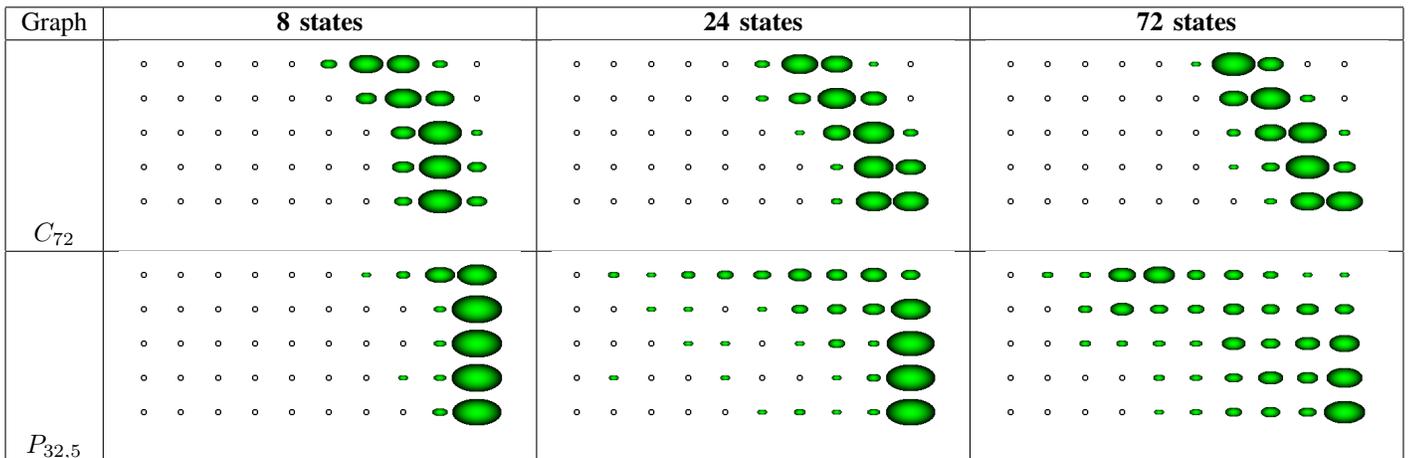| Graph	8 states	24 states	72 states
C_{72}			
$P_{32,5}$			

Fig. 11. Play profiles for 8, 24, and 72 state machines evolved on different graphs with the graphs limiting play in the $P_{32,5}$ graph and mating in the C_{36}.
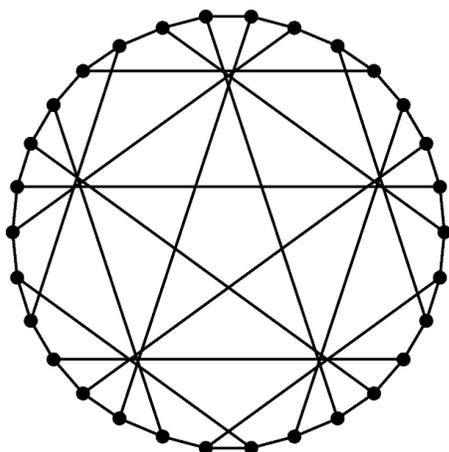
Fig. 12. The Tutte-Coxeter or Levi graph, the smallest 3-regular graph of girth eight.

of vertices, is shown in Figure 12. Another parameter that might correlate with cooperation behavior is the distribution of eccentricities of the graph. The *eccentricity* of a vertex is the largest distance to another vertex in the graph. The eccentricity distribution is an enumeration of the fractions of vertices with each possible eccentricity. This is another graph parameter that, like girth, measures how spread out the graph is. Exploring these parameters, as well as searching for others that have some explanatory power, is an early priority for future research.

B. Evolutionary Search for Cooperative Graphs

In [8], [10] a system was devised to evolve connection topologies that exhibit a particular behavior when they are used as the pattern of contacts in a population undergoing an epidemic. Another possible next step is to try to evolve graphs to match a particular play profile. A potential problem with this strategy is that the fitness evaluation is quite costly, but it has the potential to provide targeted, valuable information about which graph parameters are predictive of the impact of cooperation of a population of evolving agents. The current study demonstrates that there is something to search *for* and more limited objective functions that match to a full play profile may yield useful results.

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